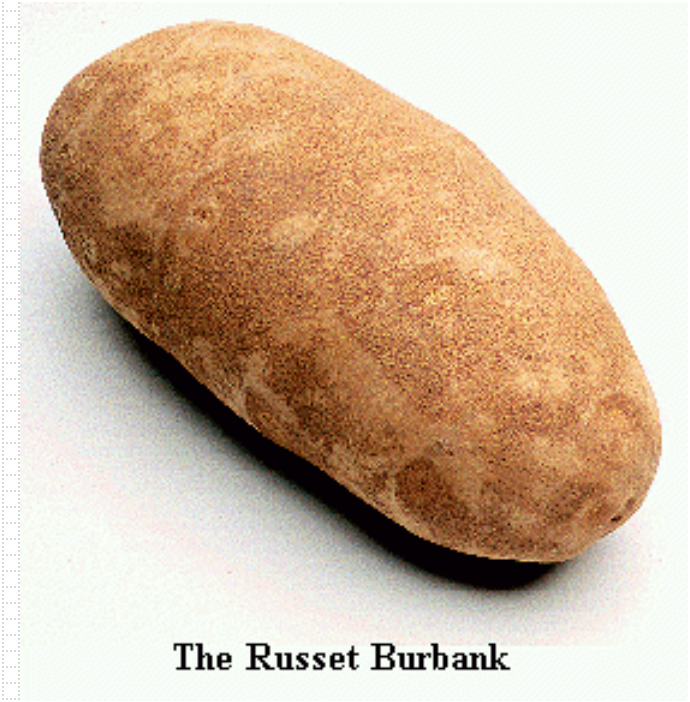# 7.1 Integral as Net Change

Example: National Potato Consumption

The rate of potato consumption
for a particular country was:

$$C(t) = 2.2 + 1.1^t$$

where *t* is the number of years
since 1970 and *C* is in millions of
bushels per year.

The Russet Burbank

Find the amount of potatoes consumed from the
beginning of 1972 to the end of 1973.

→

Example 5:   National Potato Consumption

$$C(t) = 2.2 + 1.1^t$$

To find the cumulative effect over time – Integrate it!

From the beginning of 1972 to the end of 1973:

$$\int_2^4 2.2 + 1.1^t \, dt = 2.2t + \frac{1}{\ln 1.1} 1.1^t \Big|_2^4 \approx 7.066 \quad \text{million bushels}$$

$\rightarrow$

Net Change from Data: what if we don't have the a function to work with?

Example (p. 369): A pump connected to a generator operates at a varying rate, depending on how much power is being drawn from the generator. The rate (gallons per minute) at which the pump operates is recorded at 5-minute intervals for an hour as shown in the table. How many gallons were pumped during the hour?

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

→

Time (min)	Rate (gal/min)
0	58
5	60
10	65
15	64
20	58
25	57
30	55
35	55
40	59
45	60
50	60
55	63
60	63

Gallons pumped = $\displaystyle\int_0^{60} R(t)\,dt$

We don't have a formula for $R(t)$, so we have to approximate the integral – the trapezoidal rule works well:

$$\frac{1}{2}\cdot 5\cdot\left[58+2(60)+\cdots+2(63)+63\right]$$

$$=3582.5 \text{ gallons}$$

$\rightarrow$

2010 BC1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

(a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?

$$\int_0^6 f(t)\,dt = 142.274 \text{ or } 142.275$$

2010 BC1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

(b) Find the rate of change of the volume of snow on the driveway at 8 A.M.

$\left.\begin{matrix} f(8) - g(8) \\ \text{OR} \\ f(8) - 108 \end{matrix}\right\} = -59.582 \text{ OR } -59.583$

2010 BC1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

(c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.

$$h(t) = \begin{cases} 0 & 0 \leq t < 6 \\ 125(t-6) & 6 \leq t \leq 7 \\ 125 + 108(t-7) & 7 < t \leq 9 \end{cases}$$

$$h(t) = h(6) + \int_6^t 125\, dx = 125x \Big|_6^t = 125(t-6)$$

$$h(t) = h(7) + \int_7^t 108\, dx = 125 + 108(t-7)$$

2010 BC1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. $(t = 6)$. The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \le t < 6 \\ 125 & \text{for } 6 \le t < 7 \\ 108 & \text{for } 7 \le t \le 9. \end{cases}$$

(d) How many cubic feet of snow are on the driveway at 9 A.M.?

$$\int_0^9 f(t) - g(t)\, dt$$

$$\int_0^9 f(t)\,dt - \int_0^9 g(t)\,dt = \boxed{\int_0^9 f(t)\,dt - h(9)} = 26.334 \text{ or } 26.335$$

Classwork:

Chapter 7 AP Packet #23 – 25

Homework:

Chapter 7 AP Packet #27 – 29